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Broken symmetries and excitation modes in liquid helium II

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At about absolute zero, liquid He-4 lies in its ground state consisting of bare particles and the dynamical equations obey certain symmetries. The field interaction changes the behaviour of this system to physical or quasi-particle states and the system becomes superfluid, where the above symmetry becomes broken. The most elegant way of describing this difference is to establish relationships, in the form of canonical commutation relation, which take different form when broken symmetries are said to have taken place. Here an important role is played by massless particles—the phonons, which represent the Goldstone particles.

To understand the phenomenon, these Bosons are described with vanishing momentum in terms of the linear homogeneous equation valid for a massless stable physical field, which becomes invariant under inhomogeneous transformation expressed by

$$\psi = \psi + \alpha \tag{1}$$

where α is a C-number constant, representing Bose-Einstein condensation ψ is made up of creation and annihilation operators of massless physical particles, the transformation (1) is canonical if ψ is a Bose field. The induction of C number, breaks the symmetry.

Commutation Relations. The symmetry eq. (1) leaves invariant the following canonical commutation relations of the free massless Bose field in liquid He-II.

$$[\psi(x), \pi(x')]|_{t=t'} = i\delta^3(x-x'), \quad \ldots \quad (2)$$

where

$$\pi(x) = \frac{w}{dt}\,\psi(x). \quad \ldots \quad (3)$$

Restricting to an infinitesimal α, the symmetry operation (1) can be imagined as an infinitesimal unitary transformation in Hilbert space,

Hence
$$\delta\psi(x) = \alpha = \frac{1}{i}[\psi(x), \alpha Q], \tag{4}$$

Here $$Q = \int_v \pi(x) d^3x. \qquad \dots (5)$$

From the equal time commutation relation we get

$$\frac{1}{i} [\psi(x), \alpha Q] = -i\alpha \int_v [\psi(x), \pi(x')] d^3x$$
$$= \alpha. \qquad \dots (6)$$

When the field reaction is taken into account, Bose field of liquid He II can now be written in terms of the following decomposition

$$\psi(x) = \frac{1}{\sqrt{v}} \sum_k \frac{1}{\sqrt{2|k|}} (a_k e^{ikx - i|k|t} + a_k + e^{-ikx + i|k|t}). \dots (7)$$

The relation (7) is inserted in (5) to obtain

$$Q = \int \pi(x) d^3x = \left(\frac{d}{dt}\right) \psi(x) d^3x$$
$$= \lim_{k \to 0} \frac{-i\sqrt{|k|}\, v}{2} (a_k e^{-ikt} - a_k^+ e^{ikt})$$
$$= 0.$$

This is because the above is a Hermitian field.

The canonical commutation relation now gives

$$\delta\psi(x) = \frac{1}{i} [\psi(x), \alpha Q] = 0. \qquad \dots (9)$$

As the generator Q is zero, this relation contradicts the relation (6). This contradiction between (6) and (9) is understood by the name of broken symmetries in the theory of quantised fields.

The integrand (8) is zero because, the spatial integral picks up only $k = 0$ Fourier component of the integrand. This reduces Q to zero. This particle with $k = 0$ is phonon, which breaks the symmetry of He II at low temperature, creating superfluid phenomena.

Spurious Coordinates. The Symmetry breaking phenomena in liquid He-II can be elucidated by analysing the field operators $\psi(x)$ and $\pi(x)$ in terms of their normal modes as shown below :

$$\psi(x) = \frac{1}{\sqrt{v}} \sum_k q_k(t) e^{ik \cdot x}$$

$$\pi(x) = \frac{1}{\sqrt{v}} \sum_k P_k(t) e^{-ik \cdot x} \qquad \dots (10)$$

the integral of $\pi(x)$ is proportional to the canonical momentum of $k = 0$ mode.

Thus

$$Q = \int \pi(x) d^3x = \sqrt{v}\, P_k = 0 \qquad \ldots \quad (11)$$

but

$$P_k(t) = \dot{q}_k(t).$$

By virtue of field equation for superfluid He-4, $q_{k=0}$ is constant, because of Bose-Einstein condensation. Hence we have $P_{k=0} = 0$. Thus the vanishing of the generator Q, expresses itself in the vanishing of canonical momentum for $k = 0$ mode of massless field, which is phonon. This then contradicts the relation

$$[q_{k=0}, P_{k=0}] = i. \qquad \ldots \quad (12)$$

Because of this contradiction the occupation number $n_{p=0} = a^{\dagger}{}_{p=0} a_{p=0}$ is replaced by a C number in Bogoliubov's prescription, which neglects the dynamical behaviour of the condensed state. Therefore $k = 0$ mode does not represent a true dynamical degree of freedom.

So, He II field is quantised in a gauge in which $q_{k=0}$ mode is cancelled. In this way eq. (10) takes the form

$$\psi(x) = \frac{1}{\sqrt{v}} \sum_{k \neq 0} q_k(t) e^{ik \cdot x}$$

$$\pi(x) = \frac{1}{\sqrt{v}} \sum_{k \neq 0} p_k(t) e^{-ik \cdot x} \qquad \ldots \quad (13)$$

The relation (2) is thus modified and the commutation relation between P_k's and q_k's takes the form

$$[q_k(t),\ P_k'(t)] = i\delta_{kk'} \qquad k, k' \neq 0,$$

giving

$$[\psi(x), \pi(x')] = \frac{1}{v} \sum_{k' \neq 0} \sum_{k' \neq 0} [q_k(t), p_k'(t)] e^{ik(x-x')}$$

$$= \frac{1}{v} \sum_{k \neq 0} e^{ik(x-x')}. \qquad \ldots \quad (14)$$

or

$$\Big[\ \psi(x), \pi(x')\Big]_{t=t'} = i\Big[\delta^3(x-x') - \frac{1}{v}\Big]. \qquad \ldots \quad (15)$$

Conclusion

Relation (5) follows from the fact that $\rho_{k=0}$ mode has been neglected $\rho_k = 0$ mode is taken care of by B. E. Condensation, which is represented by iv^{-1} term in eq. (15). The onset of superfluid phenomena created by phonon is mathematically visualised in infinite volume limit. When v tends to infinity, v^{-1} in (15) does not vanish, because the integral of this term over the quatisation volume remains equal to unity. When v tends to infinitry' v^{-1} term gives rise to infinitesimal amplitude at every point in space of Boson field such that its integral over all space remains unity. Such terms have been noticed in the theory of quantised field (Goldstone 1961, 1962, Umezawa 1965). This idea explains the phenomena of long range correlation in superfluid system.

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A study on discharge oscillations

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In a discharge tube operating in the negative resistance region of its V-I characteristics, the negative glow, as pointed out by Sanduloviciu (1968) is a source of self-excited oscillations.

Depending upon the particular experimental set-up in the discharge tube system, the discharge oscillations can be represented by a suitable series or a parallel resonant circuit. The series and parallel resonant frequencies are not only equal to each other but also equal to the frequency of oscillations. The elements L, R, C of the resonant circuit consist respectively of discharge inductance and resistance between the anode and the wall at the negative glow region and capacitance of this region including the wall with respect to the surroundings.

The essential difference between the elements L, R, C of the circuit representing oscillations in a discharge tube and those dealt with in an ordinary

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L, R, C circuit is that while in the latter case the elements are independent of the current flowing through the circuit, those in the former case are not so. The nature of dependence of these elements on the current is, however, not easy to foresee. The difficulty is all the more immense, since as the results of Sanduloviciu (1971) show, the resonant frequency has a pronounced dependence on the discharge current. Thus the physical aspect of the exact nature of variation of the circuit elements and the resonant frequency ω_r with the discharge current is uncertain. This explains why many of the results obtained by Sanduloviciu (1969, 1971) remained unexplained. The findings are not only interesting but also a matter of detailed study. In the present communication only a qualitative explanation of some of these findings is offered.

From an analysis of the data of Sanduloviciu, one can construct table 1.

Table 1

Discharge current $I(mA)$	1.7	1.57	1.48	Sanduloviciu (1971)
Series impedance $Z(K\Omega)$	1.0	1.05	1.25	Sanduloviciu (1971)
Discharge inductance $L(mH)$ at				
(a) Constant gas press (air) 0.123 mmHg, anode-cathode pot. V in the range 970 < V < 1250 volts	1.25	1.35	1.45	Sanduloviciu (1969)
(b) Constant anode-cathode pot. 970 volts, gas press p (air) in the range 0.137 < p < 0.177 mmHg	0.85	0.95	1.05	,,
Resonant frequency ω_r(KHz)	612	660	688	Sanduloviciu (1971)

The above table shows that ω_r, Z and L all increase with decrease in I.

Experimentally, I can be increased by either (a) increasing the anode-cathode potential at a constant gas pressure, or (b) increasing the gas pressure at a constant anode-cathode potential.

At the present state of knowledge about the mechanism of oscillations, it is difficult to formulate an adequate theory to explain the above findings. It is possible, however, to offer a qualitative explanation as follows.

When I is increased by increasing the anode-cathode potential at a constant gas pressure, the primary electron energy is increased, the secondary electron emission from the tube wall is enhanced, the wall becomes thus more positive because of which the effective potential is reduced. The value of L arising from the accumulation of charge carriers in front of the anode thus diminishes.

Again, when I is increased by increasing the pressure at a constant anode-cathode potential, the ionizing collision would be more frequent, since the data of Sanduloviciu correspond to the higher pressure side of Paschen minimum and the field near the cathode will be strengthened due to increased space charges (Von Engel 1965). The primary electron energy therefore increases and the sequence of changes is the same as in the last paragraph.

Thus, the variation of L with I, when the latter is varied by the method (a) or (b) is explained.

In order to explain the possible variation of Z and ω with I, it may be noted from the usual V-I characteristics of the glow discharge in the negative resistance region, that the greater is the discharge current, the less is the impedance of the discharge, when the circuit representing the oscillations is non-resonant. Again, under the resonant condition, the greater is the discharge current, the less is the resistance of the discharge.

The series impedance Z for an L, C, R circuit at a frequency ω, is given by

$$Z = f(R, X) = (R^2+X^2)^{\frac{1}{2}} \qquad \ldots \quad (1)$$

where the reactance

$$X = \omega L - \frac{1}{\omega C}.$$

R, X and hence Z are, as mentioned above, dependent on I. Then considering a small variation of Z, on gets from (1)

$$\frac{\partial Z}{Z} = \frac{R^2}{Z^2}\left(\frac{\partial R}{R}\right) + \frac{X^2}{Z^2}\left(\frac{\partial X}{X}\right). \qquad \ldots \quad (2)$$

From this equation, the obvious conclusion is that p.c. change in $Z \geqslant$ p.c. change in R, the equality holding for resonant condition and inequality for non-resonant condition. Let Z_1, R_1, X_1 and Z_2, R_2, X_2 denote values of the respective parameters corresponding to currents I_1 and I_2 ($I_1 > I_2$) at slightly off-resonant frequencies ω_1 and ω_2, then since, as pointed out above, $Z_2 > Z_1$ and $R_2 > R_1$,

$$X_2 - X_1 > 0,$$

or,

$$\omega_2 L_2 - \omega_1 L_1 > \frac{1}{\omega_2 C_2} - \frac{1}{\omega_1 C_1}, \qquad \ldots \quad (3)$$

where L_1, C_1, and L_2, C_2, represent the respective values of the discharge inductance and wall capacitance corresponding to the currents I_1 and I_2.

Let us now assume that the wall capacitance is unaffected by the change of discharge current so that $C_1 = C_2$

In order to test the validity of eq. (3), three cases may be possible, viz.,

(i) $\omega_1 = \omega_2$

(ii) $\omega_1 < \omega_2$

and (iii) $\omega_1 > \omega_2$.

Since from the physical standpoint it has already been established that L decreases with the increase of I, then $L_1 < L_2$ as $I_1 > I_2$. With the help of this relation ($L_1 < L_2$) the inequality (3) easily holds for case (i) and case (ii). The inequality may still hold for case (iii) only when the p.c. increase in ω is small, because the case of equality in rates of increase in ω and decrease in L is evidently ruled out.

Thus combining (i), (ii) and (iii), it can be concluded that with decrease in current, the value of ω will either remain unaffected (i) or will increase (ii) or can decrease only slightly (iii). The results of Sanduloviciu within a limited range of observations, are in agreement with the case (ii) and by extending the region of observations one can possibly obtain results which may be in agreement with the cases (i) or (iii).

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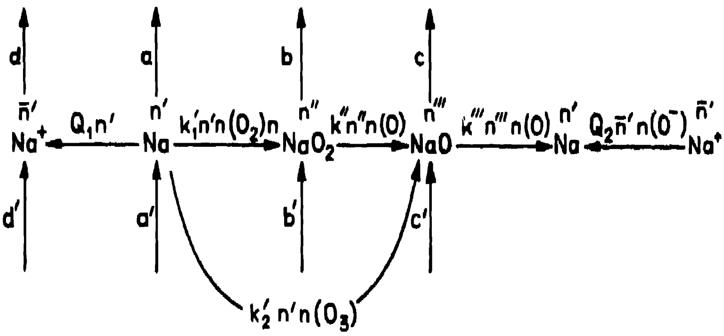
ERRATUM

Origin of Sodium at Atmosphere. S. N. Ghosh and V. Mitra [Ind. J. Phys. Vol. 48, 1 (1974)]

Add the following at the end of line 2 in page 3.

The processes which lead to the equilibrium of these constituents in any layer may be represented by the following dragram



where n, $n(O_3)$, $n(O_2)$, $n(O)$ and $n(O^-)$ are the concentrations of air, O_3, O_2, O and O^- respectively in a given layer; a and a' are the rates at which Na atoms diffuse *out* and diffuse *into* the given layer from its adjacent layers (a and a' are respectively the number of Na atoms per second per unit area which enter the given layer from the adjacent upper layer). Similarly (b, b'), (c, c') and (d, d') are the diffusion rates for NaO_2 NaO and Na respectively.